Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year		Final Term Exam Course: Mathematics 1 – A Date: January 5 , 2019 Duration: **3** hours

The Exam consists of one page   Answer **All** Questions   No. of questions: 4   Total Mark: 100

<u>Question 1</u>

(a)Complete the following : 5

(i)The sum of all eigenvalues of a unit matrix of order n equals to……………………

(ii)A square matrix A is called non singular if …….and it is called symmetric if ……

(iii)If λ is eigenvalue of a matrix A , then $\lambda+\sqrt{\lambda}$ is eigenvalue to ………….……..

(iv)A linear system AX = B is called inconsistent if ……………………………………

(v)A linear system AX = B has one solution if …………………………………………

(b)If $A=\begin{bmatrix}0 & 2 & 4\\ 1 & 3 & 0\end{bmatrix}$, $B=\begin{bmatrix}2 & -1 & 1\\ 3 & -3 & 0\end{bmatrix}$, $C=\begin{bmatrix}1 & -2 & 3\\ 2 & 0 & 1\\ 0 & -1 & 2\end{bmatrix}$ 8

Find, if possible A + B , A + C, A.B , A.C , C.A , |A| , |C|

(c)Find the eigenvalues and the eigenvectors of the matrix : $A=\begin{bmatrix}2 & 2\\ 1 & 3\end{bmatrix}$ 8

Also, Write the diagonal form and find $f(A)=\ln A$.

(d)Solve the L.S : (i) $x-2y+z=1,\ \ -2x+3y-z=0,\ \ 3x+y-2z=-3$ 2

(ii) $x-y+2z=2,\ \ -x+y-z=3,\ \ 3x-3y+4z=3$ 2

<u>Question 2</u>

(a)Find the coefficient of y^4 in the expansion of $(y^2+\frac{1}{y^2})^6$ (b)Expand $\frac{2x-3}{(x-1)(x-2)}$ 3+3

(c)If $z_1=2+2i$, $z_2=8i$. Find $(z_1.z_2)^6$, $(\frac{z_2}{z_1})^{10}$ and $\sqrt[4]{z_2}$. 4

(d)Find u and v of the function $f(z)=\cos(iz)$. 2

(e)Solve $4x^3-24x^2+48x-32=0$ if its roots a, b, c form a geometric sequence. 3

(f)Find S_n , S_{100} and S_∞ for the series : $\sum_{r=1}^{n}\frac{1}{(3r-1)(3r+2)}$ 5

(g)By Induction, prove that : 9^n-2^n is divisible by 7, $n\geq 1$. 5

<u>Question 3</u>

(a)Find $\frac{dy}{dx}$ where : (i) $y=3\sin^3 t,\ \ x=2\cos^3 t$ (ii) $e^x\sin y=e^y\cos x$ 12

(iii) $y=e^{5x^2}.(\operatorname{arcsec} x)^8.(7-\operatorname{csch} 3x)^{-12}$

(b)Find $y^{(18)}$ where $y=\frac{3-5x}{6x+8}$. 5

(c)Find the integral : $\int(8^{3x}+x^2e^{-5x^3}-\cot 4x+\frac{x^3}{\sqrt{x-5}})dx$ 8

<u>Question 4</u>

(a)If the hypotenuse of right triangle is constant. Show that its area is maximum when it is isosceles . 5

(b)Find the equation of tangent and normal for the curve: $y=\frac{(\ln x)^x}{2^{\ln x-1}}$ at x = e. 8

(c)Find the limits: (i) $\lim\limits_{x\to\infty}\frac{5^x+2^x}{9^x}$ (ii) $\lim\limits_{x\to\infty}x^{\frac{1}{\sqrt{x}}}$ (iii) $\lim\limits_{x\to\infty}e^{-x}\ln x$ (ii) $\lim\limits_{x\to 0}(\frac{1}{x}-\frac{1}{e^x-1})$ 12

Good Luck   *Dr. Khaled Elnagar*   *Dr. Mohamed Eid*   *Dr. Elsayed Gareeb*

Model Answer

Answer of Question 1

(a)(i)The sum of all eigenvalues of a unit matrix of order n equals to n.

(ii)A square matrix A is called non singular if $|A| \neq 0$ if and it is called symmetric if $A = A`$

(iii)If λ is eigenvalue of a matrix A , then $\lambda + \sqrt{\lambda}$ is eigenvalue to $A + \sqrt{A}$.

(iv)A linear system AX = B is called inconsistent if it has no solution.

(v)A linear system AX = B has one solution if rank A = rank G = number of unknowns.

--*5 Marks*

(b)If $A = \begin{bmatrix} 0 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & 1 \\ 3 & -3 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}$

A + C, A.B, C.A and |A| are nor exist.

$A + B = \begin{bmatrix} 2 & 1 & 5 \\ 4 & 0 & 0 \end{bmatrix}$, $A.C = \begin{bmatrix} 4 & -4 & 10 \\ 7 & -2 & 6 \end{bmatrix}$, $|C| = 1 + 8 - 6 = 3$

--*8 Marks*

(c) $\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda) - 2 = \lambda^2 - 5\lambda + 4 = 0$. Then $\lambda_1 = 1, \quad \lambda_2 = 4$

From the equation, $\begin{bmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

For : $\lambda_1 = 1$, $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then x + 2y = 0, x + 2y = 0

Put x = 2, then the corresponding

eigenvector is : $X_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

For : $\lambda_2 = 4$, $\begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then –2x + 2y = 0, x – y = 0

Put y = 1, we get x = 1 and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Then $T = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$ and $T^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$

The diagonal form : $T^{-1}A\,T = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$. Then $T^{-1} \ln A\ T = \begin{bmatrix} \ln 1 & 0 \\ 0 & \ln 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1.4 \end{bmatrix}$

Then $\ln A = T\begin{bmatrix} 0 & 0 \\ 0 & 1.4 \end{bmatrix}T^{-1} = \frac{1}{3}\begin{bmatrix} 1.4 & 2.8 \\ 1.4 & 2.8 \end{bmatrix}$

--*8 Marks*

(d)By Calculator, the solution is : (1, 2, 4).

(ii) $G = \left[\begin{array}{ccc:c} 1 & -1 & 2 & 2 \\ -1 & 1 & -1 & 3 \\ 3 & -3 & 4 & 3 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & -1 & 2 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & -2 & -3 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & -1 & 2 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 7 \end{array}\right]$

Rank A = 2, Rank G = 3. No solution.

--*4 Marks*

Dr. Mohamed Eid

Answer of Question 4

a) Since the hypotenuse of the right triangle is constant, therefore $x^2 + y^2 = c^2 \Rightarrow$ $2x + 2yy` = 0 \Rightarrow y` = -x/y$

Area: $A = \frac{1}{2}xy \Rightarrow \frac{dA}{dx} = \frac{1}{2}(xy`+y) = \frac{1}{2}(x(\frac{-x}{y}) + y) = 0 \Rightarrow x^2 = y^2 \Rightarrow x = y$

Therefore the area is maximum when the triangle is isosceles.

b) $y` = \dfrac{[\ell n\, x]^x [\ell n(\ell n x) + x(\frac{1/x}{\ln x})][2^{\ell n(x)-1}] - [\ell n\, x]^x [\frac{1}{x}][2^{\ell n(x)-1}]\,\ell n\, 2}{[2^{\ell n(x)-1}]^2}$

at x = e, slope of the tangent = y`(1) = $1 - \frac{\ell n\, 2}{e} = \frac{e - \ell n\, 2}{e}$

Therfore the point of contact is (e, 1) and hence the equation of tangent is $\frac{y-1}{x-e} = \frac{e - \ell n\, 2}{e}$

Also equation of normal is $\frac{y-1}{x-e} = \frac{e}{\ell n\, 2 - e}$

c-i) $\lim\limits_{x\to\infty} \frac{5^x + 2^x}{9^x} = \lim\limits_{x\to\infty} \frac{5^x(1 + (\frac{2}{5})^x)}{9^x} = \lim\limits_{x\to\infty} (\frac{5}{9})^x = 0$

ii) This limit of the indeterminate form ∞^0. Let $y = x^{1/\sqrt{x}} \Rightarrow \ell n\, y = \frac{\ell n\, x}{\sqrt{x}}$

Hence L'Hopital rule is ready to be used such that

$$\lim_{x\to\infty} \ell n\, y = \lim_{x\to\infty} \frac{\ell n\, x}{\sqrt{x}} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{1/x}{1/(2\sqrt{x})} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = 0 .$$

Therefore $\lim\limits_{x\to\infty} x^{1/\sqrt{x}} = \lim\limits_{x\to\infty} e^{\ell n\, y} = e^{\lim\limits_{x\to\infty} \ell n\, y} = e^0 = 1$

iii) $\lim\limits_{x\to 0} [\frac{1}{x} - \frac{1}{e^x - 1}] = \lim\limits_{x\to 0} [\frac{e^x - 1 - x}{x(e^x - 1)}] = \lim\limits_{x\to 0} [\frac{e^x - 1}{xe^x + (e^x - 1)}] = \lim\limits_{x\to 0} [\frac{e^x}{xe^x + 2e^x}] = \frac{1}{2}$

iv) $\lim\limits_{x\to\infty} e^{-x}\, \ell n\, x = \lim\limits_{x\to\infty} \frac{\ell n\, x}{e^x} = \lim\limits_{x\to\infty} \frac{1}{xe^x} = 0$
